

8. Solve the assignment problem

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	54	53	31	17	28

Sol<sup>n</sup>:

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	54	53	31	17	28

Step I:  
Subtracting the smallest element of each row from every element of the corresponding row

	I	II	III	IV	V	VI
A	0	13	49	2	10	18
B	0	35	29	7	20	5
C	13	0	63	9	17	5
D	47	15	0	22	12	5
E	25	0	46	11	14	7
F	0	53	50	28	14	25

Step II:  
Subtracting the smallest element of each column from every element of the corresponding column

	I	II	III	IV	V	VI
A	0	13	49	0	0	13
B	0	35	29	5	10	0
C	13	0	63	7	7	0
D	47	15	0	20	2	0
E	25	0	46	9	4	2
F	0	53	50	26	4	20

Step III:

Starting from first row, find the row having only one zero and mark it as  $\square$  and cross (X) all other zeros in the corresponding column.

Starting from first column, find the column having only one zero unmarked zero and mark it  $\square$  and cross all

	I	II	III	IV	V	VI
A	X	13	49	$\square$	X	13
B	X	35	29	5	10	$\square$
C	13	X	63	7	7	X
D	47	15	$\square$	20	2	X
E	25	$\square$	46	9	4	0
F	$\square$	53	50	26	4	20

~~Step IV:~~

Since row 3 and column 5 have no assignments, so we proceed to the next step

Step IV:

In this step, we draw minimum number of lines

	L1 I	L2 II	III	IV	V	VI	
A	X	13	49	0	X	13	L4
B	X	35	29	5	10	0	✓(5)
C	13	X	63	7	7	X	✓(0)
D	47	15	0	20	2	X	L5
E	25	0	46	9	4	2	✓(4)
F	0	53	50	26	4	20	✓(7)
	✓(4)	✓(2)				✓(3)	

- i) mark (✓) row 3 in which there is no assignment.
- 2) then mark (✓) column 2 and 6 which have zero marked row 3
- 3) mark (✓) rows 5 and 2 which have assignment in the marked column 2 & 6
- 4) mark (X) row 6 which has assignment in the column 1 & 2.
- 5) Mark column 1 (not already marked) which has zero in the marked row 2.
- 6) mark row 6 which has assignment in the marked column 1.
- 6) Now Draw lines through all marked columns 1, 2, 6, then draw lines through unmarked row 1 and 4 having zeros through which there is no line.  
Thus, we get five lines (min. no.) to cover all the zeros.

Step V:

Now, the smallest element which do not have any line = 4

Subtracting 4 from all the elements that do not have a line through them and adding to every element that lies at the intersection of two lines and leaving the remaining elements unchanged.

we do not need to consider the elements which are crossed out

	I	II	III	IV	V	VI
A	4	17	49	0	17	17
B	0	35	25	1	6	0
C	13	0	59	3	3	0
D	51	19	0	20	2	4
E	25	0	42	5	0	2
F	0	53	46	22	0	20

we have to find the minimum cost

A → IV

A → IV

B → I

B → VI

C → VI

C → II → Try to solve

D → III

D → III

E → II

E → V

F → V

F → I

↓

Min. Cost = 11 + 43 + 33 + 27 + 11 + 17

= 142

Q. Solve the following minimal assignment problem.

	2	3	4	5
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	21	24	17	28
E	14	10	12	15

Sol<sup>n</sup>.

	2	3	4	5
A	3	9	8	12
B	3	6	0	9
C	1	4	3	0
D	4	7	0	11
E	4	0	2	1

	1	2	3	4	5
A	2	9	0	8	8
B	2	1	6	0	5
C	0	4	3	X	X
D	3	7	X	11	5
E	3	0	2	1	1

	1	2	3	4	5	
A	0	7	X	6	6	A → 1
B	2	1	8	0	5	B → 4
C	X	4	5	X	0	C → 5
D	1	5	0	9	3	D → 3
E	3	0	4	1	1	E → 2

Total Cost = 11 + 6 + 16 + 17 + 10  
= 60